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THREE DIMENSIONAL MADEY'S THEOREM AND THE GENERALIZED BRIGHTNESS FUNCTION*

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We discuss the effect of electron beam emittance on the gain of low gain free-electron lasers in terms of the generalized brightness functions(GBFs). A GBF is a Wigner function of the electric field, and can be interpreted as a phase space probability distribution of the photons. For a transversely coherent radiation field such as the undulator radiation from a single electron, the phase space area described by GBF is about $\lambda/2$, where λ is the wavelength. The GBF corresponding to the undulator radiation from an electron beam is given by a convolution of the single electron GBF and the probability distribution of the electrons' phase space distribution. For a low gain free electron laser, the gain taking into account of the three dimensional effects can be expressed in a form which is a generalization of the Madey's theorem; a derivative of a convolution of three phase space distribution functions - the GBF of the undulator radiation, the GBF of the input radiation, and the electron distribution function. An analysis of the effect of electron beam emittance based on this formula is presented.

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